

$$A = \left(\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 2 & 0 & 0 & 1 & 0 \\ 2 & 2 & 2 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} \downarrow \\ \end{array}$$

$$\left(\begin{array}{ccc|ccc} 3 & 2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} -\frac{2}{3} \\ \downarrow \\ \end{array}$$

$$\left(\begin{array}{ccc|ccc} 3 & 2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & \frac{2}{3} & 2 & 0 & -\frac{2}{3} & 1 \end{array} \right) \begin{array}{l} -\frac{2}{3} \\ \downarrow \\ \end{array}$$

$$\left(\begin{array}{ccc|ccc} 3 & 2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -\frac{2}{3} & -\frac{2}{3} & 1 \end{array} \right) \begin{array}{l} \cdot \frac{1}{3} \\ \cdot \frac{1}{2} \end{array}$$

→ Rang(A) = 3

$$\left(\begin{array}{ccc|ccc} 1 & \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{2} \end{array} \right) \begin{array}{l} \downarrow \\ -\frac{2}{3} \\ \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{2} \end{array} \right) A^{-1}$$

Probe:

$$\begin{pmatrix} 0 & 1 & 0 \\ 3 & 2 & 0 \\ 2 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ 1 & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{3} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Beispiel:

$$x_1 + 2x_2 + x_3 + 2x_4 + 2x_5 + 2x_6 = 8$$

$$x_3 + 3x_4 = 8$$

$$x_5 - 3x_6 = 8$$

$$-2x_1 - 4x_2 - x_3 - x_4 - 3x_5 - 7x_6 = 0$$

Schritt 1: $(A, \underline{b}) =$

$$\left(\begin{array}{cccccc|c} 1 & 2 & 1 & 2 & 2 & 2 & 8 \\ 0 & 0 & 1 & 3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 & -3 & 8 \\ -2 & -4 & -1 & -1 & -3 & -7 & 0 \end{array} \right) \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} 2$$

Schritt 2:

$$\left(\begin{array}{cccccc|c} 1 & 2 & 1 & 2 & 2 & 2 & 8 \\ 0 & 0 & 1 & 3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 & -3 & 8 \\ -2 & -4 & -1 & -1 & -3 & -7 & 0 \end{array} \right) \begin{array}{l} \downarrow \\ \downarrow \\ \downarrow \\ \downarrow \end{array} \begin{array}{l} \\ \\ \\ 2 \end{array}$$

$$\left(\begin{array}{cccccc|c} 1 & 2 & 1 & 2 & 2 & 2 & 8 \\ 0 & 0 & 1 & 3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 & -3 & 8 \\ 0 & 0 & 1 & 3 & 1 & -3 & 16 \end{array} \right) \begin{array}{l} \downarrow \\ \downarrow \\ \downarrow \\ \downarrow \end{array} \begin{array}{l} \\ \\ \\ -1 \end{array}$$

$$\left(\begin{array}{cccccc|c} 1 & 2 & 1 & 2 & 2 & 2 & 8 \\ 0 & 0 & 1 & 3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 & -3 & 8 \\ 0 & 0 & 0 & 0 & 1 & -3 & 8 \end{array} \right) \begin{array}{l} \downarrow \\ \downarrow \\ \downarrow \\ \downarrow \end{array} \begin{array}{l} \\ \\ \\ -1 \end{array}$$

$$\left(\begin{array}{cccccc|c} 1 & 2 & 1 & 2 & 2 & 2 & 8 \\ 0 & 0 & 1 & 3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 & -3 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

→ Eine Lösung existiert.

Schritt 3:

$$\left(\begin{array}{cccccc|c} 1 & 2 & 1 & 2 & 2 & 2 & 8 \\ 0 & 0 & 1 & 3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 & -3 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \begin{array}{l} \uparrow \\ - \\ \end{array}$$

$$\left(\begin{array}{cccccc|c} 1 & 2 & 0 & -1 & 2 & 2 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 & -3 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \begin{array}{l} \uparrow \\ -2 \\ \end{array}$$

$$\left(\begin{array}{cccccc|c} 1 & 2 & 0 & -1 & 0 & 8 & -16 \\ 0 & 0 & 1 & 3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 & -3 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

zur Form B:

$$\underline{x_2 = \lambda_1}$$

$$\underline{x_4 = \lambda_2}$$

$$\underline{x_6 = \lambda_3}$$

$$\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$$

3. Zeile:

$$x_5 - 3x_6 = 8, \text{ also}$$

$$\underline{x_5 = 8 + 3x_6}$$

2. Zeile:

$$x_3 + 3x_4 = 8, \text{ also}$$

$$\underline{x_3 = 8 - 3x_4}$$

1. Zeile:

$$x_1 + 2x_2 - x_4 + 8x_6 = -16,$$

$$x_1 = -16 - 2x_2 + x_4 - 8x_6$$

$$\underline{= -16 - 2x_2 + x_4 - 8x_6}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} -16 \\ 0 \\ 8 \\ 0 \\ 8 \\ 0 \end{pmatrix} + x_1 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -8 \\ 0 \\ 0 \\ 0 \\ 3 \\ 1 \end{pmatrix}$$

$\begin{matrix} = \\ = \\ = \\ = \\ = \\ = \end{matrix}$

Schritt 4:

$$\begin{array}{cccccc|c} & \downarrow & & \downarrow & & \downarrow & \\ \begin{pmatrix} 1 & -2 & 0 & +1 & 0 & -8 & -16 \\ 0 & 0 & 1 & -3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 & +3 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Schritt 5:

$$\begin{array}{cccccc|c} & \downarrow & & \downarrow & & \downarrow & \\ \begin{pmatrix} 1 & -2 & 0 & +1 & 0 & -8 & -16 \\ 0 & 0 & 1 & -3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 & +3 & 8 \end{pmatrix} \end{array}$$

Schritt 6

$$\begin{array}{cccccc|c} \begin{pmatrix} 1 & -2 & 0 & +1 & 0 & -8 & -16 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & +3 & 8 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Schritt 7:

$$\left(\begin{array}{ccc|c} -2 & +7 & -8 & -16 \\ 1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & +3 & 8 \\ 0 & 0 & 1 & 0 \end{array} \right)$$

$\begin{array}{cccc} \parallel & \parallel & \parallel & \parallel \\ c_1 & c_2 & c_3 & d \end{array}$

$$\text{Lös}^u(A, \underline{b}) =$$

$$\begin{pmatrix} -16 \\ 0 \\ 8 \\ 0 \\ 8 \\ 0 \\ 0 \end{pmatrix} + \text{span} \left(\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 7 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -8 \\ 0 \\ 0 \\ 0 \\ 3 \\ 1 \\ 0 \end{pmatrix} \right)$$

$\begin{array}{ccc} \parallel & \parallel & \parallel \\ c_1 & c_2 & c_3 \end{array}$

$$\left(\begin{aligned} \dim(\text{Lös}(A)) &= \text{Anzahl der Unbekannten} \\ &\quad - \text{Rang}(A) \\ &= 6 - 3 \\ &= 3 \quad \checkmark \end{aligned} \right)$$